

On Fixed-multiplicity Corrections to Correlators

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Correction terms generated in the correlator analysis due to multiplicity-dependent observable mean are investigated. A procedure for subtraction of such terms from calculated correlator estimates is suggested and the obtained results are discussed.

I. INTRODUCTION.

Behavior of strongly interacting matter under extreme conditions attracts interest of physicists for a considerably long time. Motivated partially also by attempts to explain interesting cosmic-ray observations [1] it has been assumed, that in relativistic collisions of hadrons or nuclei a short-living system containing a super-dense thermally equilibrated matter can be created in laboratory. Intriguing results of experimental groups at CERN and BNL show us that such an effort is a challenging task and interpretations of the obtained experimental information are still vividly discussed.

It has been suggested [2] that calculation of the correlator ratios may allow us to estimate quantitatively a degree of thermalization of high-multiplicity thermodynamical systems created in relativistic collisions of hadrons or nuclei. This suggestion has been met with a considerable interest [3] and recently a procedure for the evaluation of higher-order correlators in a reasonable computer time has been presented [4].

On experimental side correlator $C_2^{p_t}$ has been evaluated using transverse momenta of charged particles created in relativistic $Au+Au$ collisions at RHIC [5] and centrality dependence of such correlator has been studied. However, it seems that calculation of correlators from real experimental data requires a more detailed approach. For example, if the observable mean of quantity (e.g. $\langle p_t \rangle$) used for calculation of correlators depends on measured multiplicity, correlators can be systematically shifted and obtained results biased.

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In the following sections we study quantitatively the influence of multiplicity-dependent observable mean on correlators and describe a simple procedure allowing one to subtract the introduced systematical bias from the calculated correlator estimates.

II. A SIMPLE CORRECTION TO C_2 CORRELATOR

Let us start with global 2-particle correlator $C_2^{(n,x)}$ calculated from measured characteristics x_i (e.g. transverse momentum p_t) of observed charged particles with multiplicity n . For the sample A containing N_{ev} events one has (see Eq.(21) in [4] and also Eq.(2) in [5])

$$C_2^{(n,x)} = \frac{1}{N_{ev}} \sum_{N_{ev}} \sum_{i \neq j}^{[N_{ij}]} \frac{(x_i - \langle \bar{x}_G \rangle)(x_j - \langle \bar{x}_G \rangle)}{N_{ij}} \quad (1)$$

where $N_{ij} = n!/(n-2)!$ is the number of all particle pairs in a given event with indexes $i \neq j$ (symbol $[N_{ij}]$ reminds double-sum) and $\langle \bar{x}_G \rangle$ is the global mean of particle characteristics x_i (e.g. transverse momentum p_t) over all events and particles used for the analysis.

Because correlators are not sensitive [4] to number of particles n selected for their calculation (see Section 3 and Tab.3 in [4]) one can select a fixed number of tracks $n < n_k$ for the correlator evaluation in the sample of events with multiplicities $n_1 < n_k < n_2$.

It can be easily shown that if observable mean $\bar{x}(n_k)$ in events with observed multiplicity n_k depends on event multiplicity n_k then calculated value of correlator C_2 for sample A is shifted by the amount Δ_2 which depends on the width of multiplicity interval (n_1, n_2) and on properties of multiplicity-dependent observable mean $\bar{x}(n_k)$.

$$\text{if } \frac{d\bar{x}(n)}{dn} \neq 0 \quad ; \quad C_2^{\text{calc}} = C_2^{\text{true}} + \Delta_2 \quad (2)$$

In order to demonstrate this let us divide event sample A into subsamples $A1$ and $A2$ with multiplicities (n_1, n_{1a}) and (n_{1a}, n_2) and let us assume (see Fig.1) that observable mean $\bar{x}(n_k)$ increases with multiplicity n_k in step-like way: $\bar{x}(n_1 \leq n_k \leq n_{1a}) = \bar{x}_{A1}$ and $\bar{x}(n_{1a} < n_k \leq n_2) = \bar{x}_{A2}$. For correlator $C_2^{(n,x)}$ calculated for event sample A one can write

$$C_2^{A=A1+A2} = \frac{1}{N_{ev}} \left[\sum_{N_{A1}} \sum_{i \neq j}^{[N_{ij}]} \frac{(x_i - \langle \bar{x}_G \rangle)(x_j - \langle \bar{x}_G \rangle)}{N_{ij}} + \sum_{N_{A2}} \sum_{i \neq j}^{[N_{ij}]} \frac{(x_i - \langle \bar{x}_G \rangle)(x_j - \langle \bar{x}_G \rangle)}{N_{ij}} \right] \quad (3)$$

It is clear that in subsample $A1$ with observable mean \bar{x}_{A1} the correlator C_2 is calculated using an overestimated global mean $\langle \bar{x}_G \rangle$ which introduces a static shift of global mean

$\Delta\bar{x}_{A1} = \bar{x}_{A1} - \langle\bar{x}_G\rangle$ in the correlator calculation. Since $(x_i - \langle\bar{x}_G\rangle) = (x_i - \bar{x}_{A1} + \Delta\bar{x}_{A1})$ one obtains for events within sub-sample A1 (and similarly A2):

$$(x_i - \langle\bar{x}_G\rangle)(x_j - \langle\bar{x}_G\rangle) = (x_i - \bar{x}_{A1})(x_j - \bar{x}_{A1}) + \Delta\bar{x}_{A1}[x_i - \bar{x}_{A1} + x_j - \bar{x}_{A1}] + (\Delta\bar{x}_{A1})^2 \quad (4)$$

A substitution into Eq.(3) using $\sum_{N_{Ak}} \sum_{i=1}^n (x_i - \bar{x}_{Ak}) = 0$ (for $k=1,2$) gives

$$C_2^{A1+A2} = \frac{N_{A1} \cdot C_2^{A1} + N_{A2} \cdot C_2^{A2}}{N_{ev}} + \frac{N_{A1} \cdot (\Delta\bar{x}_{A1})^2}{N_{ev}} + \frac{N_{A2} \cdot (\Delta\bar{x}_{A2})^2}{N_{ev}} \quad (5)$$

where the correlator for sub-sample A1 (and similarly for A2) is

$$C_2^{A1} = \frac{1}{N_{A1}} \sum_{N_{A1}} \sum_{i \neq j}^{[N_{ij}]} \frac{(x_i - \bar{x}_{A1})(x_j - \bar{x}_{A1})}{N_{ij}}. \quad (6)$$

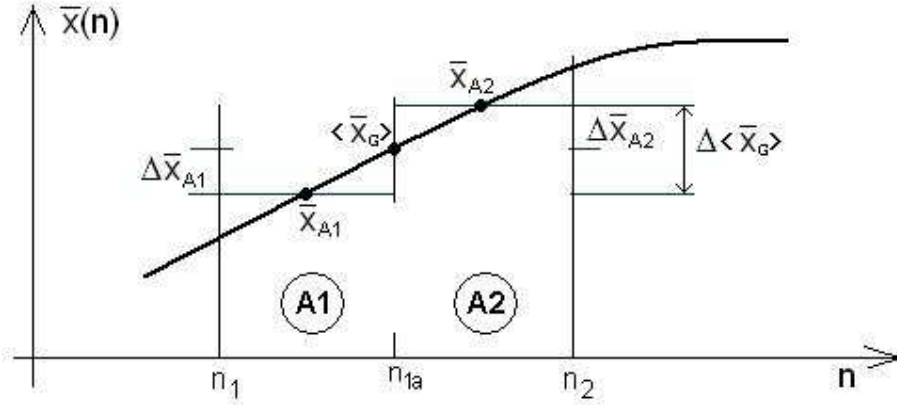


Fig.1: Calculation of C_2 correlator using sub-samples A1 and A2.

Assuming $N_{A1} = N_{A2} = N_{ev}/2$ and $|\Delta\bar{x}_{A2}| = |\Delta\bar{x}_{A1}|$ (see Fig.1) one obtains a simple relation

$$C_2^{A1+A2} = \frac{C_2^{A1} + C_2^{A2}}{2} + \frac{(\Delta\langle\bar{x}_G\rangle)^2}{4} \quad (7)$$

where $\Delta\langle\bar{x}_G\rangle = \bar{x}_{A2} - \bar{x}_{A1} = 2\Delta\bar{x}_{A2}$. If dynamics of particle production in sub-samples A1 and A2 is identical then $C_2^{A1} \approx C_2^{A2}$ and one can express the exact "true" correlator $C_2^{\text{true}} = (C_2^{A1} + C_2^{A2})/2$ for the event sample A as

$$C_2^{\text{true}} = C_2^{\text{calc}} - \frac{(\Delta\langle\bar{x}_G\rangle)^2}{4} \quad (8)$$

Correction term $(\Delta\langle\bar{x}_G\rangle)^2/4$ can be calculated and subtracted from the correlator C_2^{calc} which is evaluated using global observable mean $\langle\bar{x}_G\rangle$ of the whole sample $A = A1 + A2$.

Increasing the number of sub-samples A_1, A_2, \dots, A_N gives (in general) a more precise correlator estimate. Using 4 subsamples of the event sample $A = A_1 + A_2 + A_3 + A_4$ one obtains

$$C_2^A = \frac{N_{A1} \cdot C_2^{A1} + N_{A2} \cdot C_2^{A2} + N_{A3} \cdot C_2^{A3} + N_{A4} \cdot C_2^{A4}}{N_{A1} + N_{A2} + N_{A3} + N_{A4}} + \frac{(\Delta \langle \bar{x}_G \rangle)^2}{4} [1 + 1/4] \quad (9)$$

if multiplicity dependence of observable mean $\bar{x}(n_k)$ increases in 4 steps: $\bar{x}(n_k \in A_1) = \bar{x}_{A1}$, $\bar{x}(n_k \in A_2) = \bar{x}_{A2}$, $\bar{x}(n_k \in A_3) = \bar{x}_{A3}$, $\bar{x}(n_k \in A_4) = \bar{x}_{A4}$ (see Fig.2).

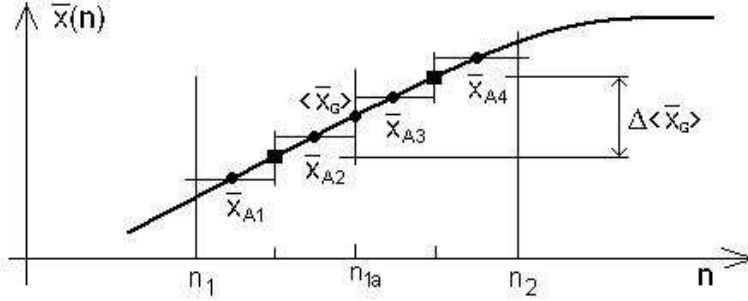


Fig.2: Calculation of C_2 correlator using subsamples $A_1 + A_2 + A_3 + A_4$.

The goal of the next section is to show how to evaluate correction terms for any given dependence of observable mean $\bar{x}_G(n_k)$ on observed event multiplicity n_k .

III. CORRECTIONS TO C_2 IN GENERAL CASE

Let us consider a general multiplicity-dependence of observable mean $\bar{x}(n_k)$. After substitution $\langle \bar{x}_G \rangle \rightarrow \langle \bar{x}_G \rangle - \bar{x}(n_k) + \bar{x}(n_k)$ in Eq.(1) correlator C_2^A is

$$C_2^A = C_2^{\text{calc}} = \frac{1}{N_{ev}} \sum_{N_{ev}} \sum_{i \neq j}^{[N_{ij}]} \frac{[x_i - \bar{x}(n_k) + \Delta \bar{x}_G(n_k)][x_j - \bar{x}(n_k) + \Delta \bar{x}_G(n_k)]}{N_{ij}} \quad (10)$$

where $\Delta \bar{x}_G(n_k) = \bar{x}(n_k) - \langle \bar{x}_G \rangle$. Here observable mean $\bar{x}(n_k)$ of events with measured multiplicity n_k is defined as

$$\bar{x}(n_k) = \sum_{N=1}^{N_{n_k}} \left[\sum_{i=1}^n \frac{x_i}{n} \right] / N_{n_k} \quad (11)$$

where N_{n_k} is total number of events in sample A with measured multiplicity n_k and n denotes number of particles (in each event) used for the calculation of correlators. Since $\sum_{N=1}^{N_{n_k}} \sum_{i=1}^n (x_i - \bar{x}(n_k)) = 0$ the expression becomes

$$C_2^{\text{calc}} = \frac{1}{N_{ev}} \sum_{N_{ev}} \sum_{i \neq j}^{[N_{ij}]} \frac{(x_i - \bar{x}(n_k))(x_j - \bar{x}(n_k))}{N_{ij}} + \sum_{N_{ev}} \frac{(\Delta \bar{x}_G(n_k))^2}{N_{ev}} \quad (12)$$

The first term on the right-hand side of Eq.(12) is a correlator calculated using a multiplicity-adjusted observable mean $\bar{x}(n_k)$. We call this "true" correlator C_2^{true} which coincides formally with global correlator defined by Eq.(1) evaluated for events with fixed multiplicity n_k . Such correlator does not contain correction terms of type given by Eq.(7) due to zero width ($n_2 = n_1 = n_k$) of multiplicity interval of events being analyzed. True correlator

$$C_2^{\text{true}} = \frac{1}{N_{ev}} \sum_{N_{ev}} \sum_{i \neq j}^{[N_{ij}]} \frac{(x_i - \bar{x}(n_k))(x_j - \bar{x}(n_k))}{N_{ij}} \quad (13)$$

is also a limiting case of the first term on the right-hand side of Eq.(9):

$$C_2^{\text{true}} = \lim_{k \rightarrow \Delta n} \frac{\sum_{i=1}^k N_i \cdot C_2^{Ai}}{\sum_{i=1}^k N_i} \quad (14)$$

where $\Delta n = n_2 - n_1$ is the maximal number of multiplicity sub-intervals in the sample A . Using a more explicit notation one has

$$C_2^{\text{true}}(\bar{x}(n_k)) = C_2^{\text{calc}}(\langle \bar{x}_G \rangle) - \sum_{N_{ev}} \frac{[\bar{x}(n_k) - \langle \bar{x}_G \rangle]^2}{N_{ev}} \quad (15)$$

where correction term $\sum (\bar{x}(n_k) - \langle \bar{x}_G \rangle)^2 / N_{ev}$ can be further expressed as

$$\sum_{N_{ev}} \frac{[\bar{x}(n_k) - \langle \bar{x}_G \rangle]^2}{N_{ev}} = \sum_{n_k=n_1}^{n_2} \frac{N_{n_k} [\bar{x}(n_k) - \langle \bar{x}_G \rangle]^2}{N_{ev}} = \int_{n_1}^{n_2} P(n_k) \cdot [\bar{x}(n_k) - \langle \bar{x}_G \rangle]^2 dn_k \quad (16)$$

Here N_{n_k} is the number of events with multiplicity n_k in sample A and probability density $P(n_k)$ for events with multiplicity n_k is $P(n_k) = N_{n_k} / \sum N_{n_k}$. Based on Eq.(15) one can find "true" correlator for the event sample A by subtracting the correction term defined by Eq.(16) from global correlator $C_2^{\text{calc}} = C_2^A(\langle \bar{x}_G \rangle)$ (calculated using the global observable mean $\langle \bar{x}_G \rangle = \sum \bar{x}_G(n_k) N_{n_k} / N_{ev}$). To summarize our result in analytical form we express "true" 2-particle correlator as

$$C_2^{\text{true}}(\bar{x}(n_k)) = C_2^{\text{calc}}(\langle \bar{x}_G \rangle) - \int_{n_1}^{n_2} P(n_k) \cdot [\bar{x}(n_k) - \langle \bar{x}_G \rangle]^2 dn_k \quad (17)$$

where meaning of $P(n_k)$, $\langle \bar{x}_G \rangle$ and $\bar{x}(n_k)$ has been described in the text above. Assuming linear multiplicity dependence of observable mean ($\bar{x}(n_k) = x_0 + \tilde{k} \cdot n_k$) and constant $P(n_k)$ distribution one evaluates integral in Eq.(17) as $\tilde{k}^2(n_2 - n_1)^2 / 12 = (\Delta \langle \bar{x}_G \rangle)^2 / 3$. This is in agreement with Eq.(9) suggesting corrections $[1 + 1/4 + 1/16 + \dots](\Delta \langle \bar{x}_G \rangle)^2 / 4$ if number of subsamples $\{A_N\}$ is iteratively doubled. (Note that $\sum_{n=0}^{\infty} 1/2^{2n} = 4/3$.)

IV. CORRECTIONS FOR C_K CORRELATORS

Results obtained for C_2 correlator can be generalized for higher order correlators. Let us define K -th order global correlator C_K

$$C_K^{(n,x)}(\langle \bar{x}_G \rangle) = \frac{1}{N_{ev}} \sum_{N_{ev}} \sum_{i_1 \neq i_2 \dots \neq i_K}^{[N_{i_1 i_2 \dots i_K}]} \frac{\prod_{m=1}^K (x_{i_m} - \langle \bar{x}_G \rangle)}{N_{i_1 i_2 \dots i_K}} \quad (18)$$

where $N_{i_1 i_2 \dots i_K} = n!/(n-K)!$ is the number of particle K -plets $\{i_1, i_2 \dots i_K\}$ made from $n < n_k$ particles in each event (n_k is total multiplicity of a given event). After substitution $\langle \bar{x}_G \rangle = \bar{x}(n_k) - \Delta \bar{x}_G(n_k)$ (where $\Delta \bar{x}_G(n_k) = \bar{x}(n_k) - \langle \bar{x}_G \rangle$) a compact expression can be found

$$C_K^{\text{calc}}(\langle \bar{x}_G \rangle) = \sum_{\lambda=0}^K \sum_{N_{ev}} \frac{(\Delta \bar{x}_G(n_k))^{K-\lambda} C_\lambda^{\text{true}}(\bar{x}(n_k))}{N_{ev}} \binom{K}{\lambda} \quad (19)$$

where $C_\lambda^{\text{true}}(\bar{x}(n_k))$ are λ -th order "true" correlators (defining $C_0^{\text{true}}(\bar{x}(n_k)) = 1$) formally obtained by replacing $\langle \bar{x}_G \rangle$ in Eq.(18) by $\bar{x}(n_k)$. In particular, for $N = 3$ one has

$$C_3^{\text{calc}}(\langle \bar{x}_G \rangle) = C_3^{\text{true}}(\bar{x}(n_k)) + 3 \sum_{N_{ev}} \frac{\Delta \bar{x}(n_k) \cdot C_2^{\text{true}}(\bar{x}(n_k))}{N_{ev}} + \sum_{N_{ev}} \frac{(\Delta \bar{x}(n_k))^3}{N_{ev}} \quad (20)$$

since $C_1^{\text{true}}(\bar{x}(n_k)) = 0$ due to definition of $\bar{x}(n_k)$. For constant probability $P(n_k)$ and for function $\Delta \bar{x}_G(n_k)$ antisymmetric around average $\langle n_k \rangle$ term $(\Delta \bar{x}_G(n_k))^3$ becomes zero. Moreover, if $C_2^{\text{true}}(\bar{x}(n_k))$ is a constant function of n_k one obtains $C_3^{\text{calc}}(\langle \bar{x}_G \rangle) = C_3^{\text{true}}(\bar{x}(n_k))$.

Under the same assumptions for $N = 4$ one has

$$C_4^{\text{calc}}(\langle \bar{x}_G \rangle) = C_4^{\text{true}}(\bar{x}(n_k)) + \sum_{N_{ev}} \left[\frac{(\Delta \bar{x}_G(n_k))^4}{N_{ev}} + 6 \frac{(\Delta \bar{x}_G(n_k))^2 C_2^{\text{true}}(\bar{x}(n_k))}{N_{ev}} \right] \quad (21)$$

and for $N = 5$

$$C_5^{\text{calc}}(\bar{x}(n_k)) = C_5^{\text{true}}(\langle \bar{x}_G \rangle) + 10 \sum_{N_{ev}} \frac{(\Delta \bar{x}_G(n_k))^2 C_3^{\text{true}}(\bar{x}(n_k))}{N_{ev}} \quad (22)$$

where $C_\lambda^{\text{true}}(\bar{x}(n_k))$ are the λ -th order true correlators defined as

$$C_\lambda^{\text{true}}(\bar{x}(\tilde{n}_k)) = \frac{1}{N_{ev}} \sum_{N_{ev}} \sum_{i_1 \neq i_2 \dots \neq i_\lambda}^{[N_{i_1 i_2 \dots i_\lambda}]} \frac{\prod_{m=1}^\lambda (x_{i_m} - \bar{x}(\tilde{n}_k))}{N_{i_1 i_2 \dots i_\lambda}} \quad (23)$$

We have thus obtained expressions for differences between $C_K^{\text{calc}} = C_K^{\text{calc}}(\langle \bar{x}_G \rangle)$ correlators calculated using global observable mean $\langle \bar{x}_G \rangle$ and true correlators $C_K^{\text{true}} = C_K^{\text{true}}(\bar{x}(n_k))$ evaluated using multiplicity-adjusted observable mean $\bar{x}(n_k)$:

$$C_K^{\text{calc}} = C_K^{\text{true}} + \Delta_K \quad (24)$$

Correction term Δ_2 according to Eq.(17) is

$$\Delta_2 = \sum_{N_{ev}} \frac{(\Delta \bar{x}_G(n_k))^2}{N_{ev}} = \int_{n_1}^{n_2} P(n) [\bar{x}(n) - \langle \bar{x}_G \rangle]^2 dn \quad (25)$$

and from Eq.(21) one has

$$\Delta_4 = \int_{n_1}^{n_2} P(n) [\bar{x}(n) - \langle \bar{x}_G \rangle]^4 dn + 6 \int_{n_1}^{n_2} P(n) C_2^{\text{true}}(\bar{x}(n)) [\bar{x}(n) - \langle \bar{x}_G \rangle]^2 dn \quad (26)$$

For $P(n_k)$ symmetrical around $\langle n_k \rangle$ term $\Delta_3 \rightarrow 0$. However, correction Δ_5 remains non-zero

$$\Delta_5 = 10 \int_{n_1}^{n_2} P(n) C_3^{\text{true}}(\bar{x}(n)) [\bar{x}(n) - \langle \bar{x}_G \rangle]^2 dn \quad (27)$$

For probability distribution $\mathcal{P}(n_k)$ *asymmetrical* around $\langle n_k \rangle$ one obtains from Eq.(20)

$$\Delta_3 = \int_{n_1}^{n_2} \mathcal{P}(n) [\bar{x}(n) - \langle \bar{x}_G \rangle]^3 dn + \int_{n_1}^{n_2} \mathcal{P}(n) C_2^{\text{true}}(\bar{x}(n)) [\bar{x}(n) - \langle \bar{x}_G \rangle] dn \quad (28)$$

In a real calculation one does not have to calculate explicitly the multiplicity-dependent observable mean $\bar{x}(n_k)$ for each multiplicity n_k . It is enough to split event sample A into reasonable number of subsamples A_1, A_2, \dots, A_N and calculate mean $\bar{x}[A_N]$ for subsamples A_N . Fitting $\bar{x}[A_N]$ values with smooth function $\bar{X}(n_k)$ gives approximation for $\bar{x}(n_k)$.

One can evaluate "true" correlators also directly using multiplicity-adjusted observable mean $\bar{x}(n_k)$ for subsets of events with fixed multiplicities n_k . This method has been utilized in p_t correlations analysis [5]. However, there is still another systematical effect which should be accounted for if one wants to obtain correlator values free from contributions due to multiplicity-dependent observable mean $\bar{x}(n_k)$. We discuss this issue in the next section.

V. FIXED-MULTIPLICITY CORRECTIONS TO CORRELATORS

Let us consider now the situation when multiplicity of particles n_k in a given event is not known precisely and instead of n^{tot} (total number of particles produced in a given event) a multiplicity of tracks $\tilde{n}_k \approx n^{\text{tot}}/\xi$ is measured in a detector. In events with a given fixed measured multiplicity \tilde{n}_k there will be fluctuations of corresponding n_k^{tot} around the average value $\langle n^{\text{tot}} \rangle \approx \tilde{n}_k \cdot \xi$ (where $\xi > 1$ is a real number corresponding to the detector acceptance). If observable mean of quantity under study does not depend on multiplicity ($\bar{x}(n) = \text{const}$) there is no influence on correlator values from these fluctuations.

However, if observable mean $\bar{x}(n)$ depends on multiplicity additional corrections $\tilde{\Delta}_n$ to calculated correlator values appear due to n_i^{tot} fluctuations at fixed measured \tilde{n}_k (which generate fluctuations of observable mean $\bar{x}(n_i^{\text{tot}})$ values at given \tilde{n}_k).

Contributions $\tilde{\Delta}_n$ are the *fixed-multiplicity* corrections to correlators. They can influence results and interpretations of the correlator analysis if they are not accounted for.

For large enough measured multiplicities \tilde{n}_k fluctuations of measured multiplicity \tilde{n}_k at given fixed total multiplicity n^{tot} are close to Gaussian (see Appendix) with probability distribution $P(\tilde{n}_k|n^{\text{tot}})$:

$$P(\tilde{n}_k|n^{\text{tot}}) = \frac{e^{-(\tilde{n}_k - \langle \tilde{n}_k \rangle)^2 / 2\sigma_{\tilde{n}_k}^2}}{\sqrt{2\pi\sigma_{\tilde{n}_k}^2}} \quad (29)$$

where $\langle \tilde{n}_k \rangle \approx n^{\text{tot}}/\xi$ and $\sigma_{\tilde{n}_k} = c \cdot \sqrt{(n^{\text{tot}}/\xi)}$ (see Appendix). One can express probability of n^{tot} fluctuations $P(n^{\text{tot}}|\tilde{n}_k)$ at given fixed measured \tilde{n}_k using Bayes' theorem [6]. For constant $P(n^{\text{tot}}) = \text{const}$ probability distribution one obtains (see Appendix)

$$P(n^{\text{tot}}|\tilde{n}_k) = \frac{e^{-(n^{\text{tot}} - \tilde{n}_k \cdot \xi)^2 / 2\sigma_{\text{tot}}^2}}{\sqrt{2\pi\sigma_{\text{tot}}^2}} \quad (30)$$

where $\sigma_{\text{tot}} = \xi \cdot \sigma_{\tilde{n}_k}$. Assuming linear approximation $\bar{x}(n^{\text{tot}}) = x_0 + \tilde{k} \cdot n^{\text{tot}}$ for multiplicity-dependent observable mean $\bar{x}(n^{\text{tot}})$ in $3\sigma_{\text{tot}}$ vicinity of $n^{\text{tot}} \approx \tilde{n}_k \cdot \xi$ one obtains fixed-multiplicity correction term $\tilde{\Delta}_2$ from Eq.(25) as

$$\tilde{\Delta}_2 = \int P(n^{\text{tot}}|\tilde{n}_k) [\bar{x}(n^{\text{tot}}) - \bar{x}(\langle n^{\text{tot}} \rangle_{\tilde{n}_k})]^2 dn^{\text{tot}} = \tilde{k}^2 \int \frac{(n - \xi \cdot \tilde{n}_k)^2 e^{-(n - \xi \cdot \tilde{n}_k)^2 / 2\sigma_{\text{tot}}^2}}{\sqrt{2\pi\sigma_{\text{tot}}^2}} dn \quad (31)$$

This gives a simple result

$$\tilde{\Delta}_2 = \tilde{k}^2 \cdot \sigma_{\text{tot}}^2(\xi) \quad (32)$$

where $\sigma_{\text{tot}}(\xi) = \sigma_{\tilde{n}_k} \xi = \xi \sqrt{\tilde{n}_k} \langle \tilde{\sigma} \rangle$ (see Appendix). For slope parameter k obtained from approximation $\bar{x}(\tilde{n}_k) = x_0 + k \cdot \tilde{n}_k$ (measured experimentally) one has $\tilde{k} = k/\xi$ and thus $\tilde{\Delta}_2 = k^2 \tilde{n}_k \langle \tilde{\sigma} \rangle^2$ where $\langle \tilde{\sigma} \rangle \approx 1$ is to be obtained from MC simulation. Using similar arguments correction term $\tilde{\Delta}_4$ can be calculated from Eq.(26) as

$$\tilde{\Delta}_4 = 3 \tilde{k}^4 \cdot \sigma_{\text{tot}}^4(\xi) + 6 \tilde{k}^2 \cdot \sigma_{\text{tot}}^2(\xi) C_2^{\text{true}} \quad (33)$$

Acceptance parameter ξ disappears ($\tilde{k} = k/\xi$ and $\sigma_{\text{tot}} = \sigma_{\tilde{n}_k} \xi$): $\tilde{\Delta}_4 = 3 k^4 \sigma_{\tilde{n}_k}^4 + 6 k^2 \sigma_{\tilde{n}_k}^2 C_2^{\text{true}}$ (where $\sigma_{\tilde{n}_k} = \sqrt{\tilde{n}_k} \langle \tilde{\sigma} \rangle$ see Appendix). Fixed-multiplicity correction terms thus depend only on experimentally measurable quantities.

One might be tempted to imply $\tilde{\Delta}_3 \rightarrow 0$ based on symmetrical probability distribution given by Eq.(30). However, fluctuations of $(n_i^{\text{tot}} - \langle n^{\text{tot}} \rangle)$ values can be significantly asymmetrical for \tilde{n}_k small enough (see Fig.3 for n_i^{tot} fluctuations at $\tilde{n}_k = 10$).

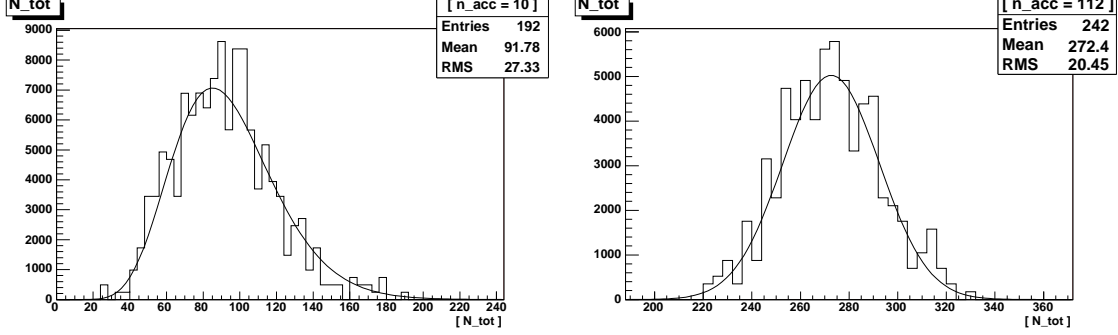


Fig.3: Fluctuations of n_i^{tot} for $\tilde{n}_k=10$, $\xi=8.51$ and $\tilde{n}_k=112$, $\xi=2.44$ (MC simulation).

In this case correction $\tilde{\Delta}_3$ can be evaluated using Eq.(28). Symmetrical $P(n^{\text{tot}}|\tilde{n}_k)$ given by Eq.(29) yields $\tilde{\Delta}_3 = 0$ and from Eq.(27) one obtains $\tilde{\Delta}_5 = 10 k^2 \sigma_{\tilde{n}_k}^2 C_3^{\text{true}}$.

VI. MC SIMULATIONS

In order to verify behavior of $\sigma_{\text{tot}}(\xi)$ and $\sigma_{\tilde{n}_k}(\tilde{n}_k)$ a simple MC simulation has been performed: Events with total multiplicities $n_k^{\text{tot}} \in (50, 2000)$ have been generated with constant probability $P(n^{\text{tot}}) = \text{const}$. In each event, rapidities y_i were assigned to n_k^{tot} particles according to the bell-shaped rapidity distribution and number of observed particles \tilde{n}_k found in the selected acceptance range $(-y_a, y_a)$ has been determined.

Two-dimensional histogram $H_2(\tilde{n}_k, n_k^{\text{tot}})$ filled with pairs of obtained numbers n_k^{tot} and \tilde{n}_k is shown in Fig.4. Projection histograms $H_{n_k^{\text{tot}}}(\tilde{n}_k)$ and $H_{\tilde{n}_k}(n_k^{\text{tot}})$ which are proportional to probabilities $P(\tilde{n}_k|n^{\text{tot}})$ from Eq.(48) and $P(n^{\text{tot}}|\tilde{n}_k)$ given by Eq.(51) are shown for $\xi \approx 4.26$.

Widths $\sigma_{\text{tot}}(\xi, \tilde{n}_k^*)$ and $\sigma_{\tilde{n}_k}(n_*^{\text{tot}} = \xi \cdot n_k^*)$ have been obtained from Gaussian fits of the projection histograms. Width $\sigma_{\tilde{n}_k}$ has been found to follow $c\sqrt{\tilde{n}_k}$ dependence with $c \approx 0.9$ in agreement with Eq.(49). Ratio of widths $\sigma_{\text{tot}}(\xi, \tilde{n}_k^*)$ and $\sigma_{\tilde{n}_k}(n_*^{\text{tot}})$ at $n_*^{\text{tot}} \approx \xi \cdot \tilde{n}_k^*$ appeared to be constant and equal to $\sigma_{\text{tot}}(\xi, n_*^{\text{tot}}/\xi)/\sigma_{\tilde{n}_k}(n_*^{\text{tot}}) \approx \xi$ as expected from Eq.(51).

Simulations for $P(n^{\text{tot}}) \neq \text{const}$ have also been done and shift $\delta n^{\text{tot}} = \langle n^{\text{tot}} \rangle_{\tilde{n}_k} - \tilde{n}_k \cdot \xi$ (described in the next section) has been observed.

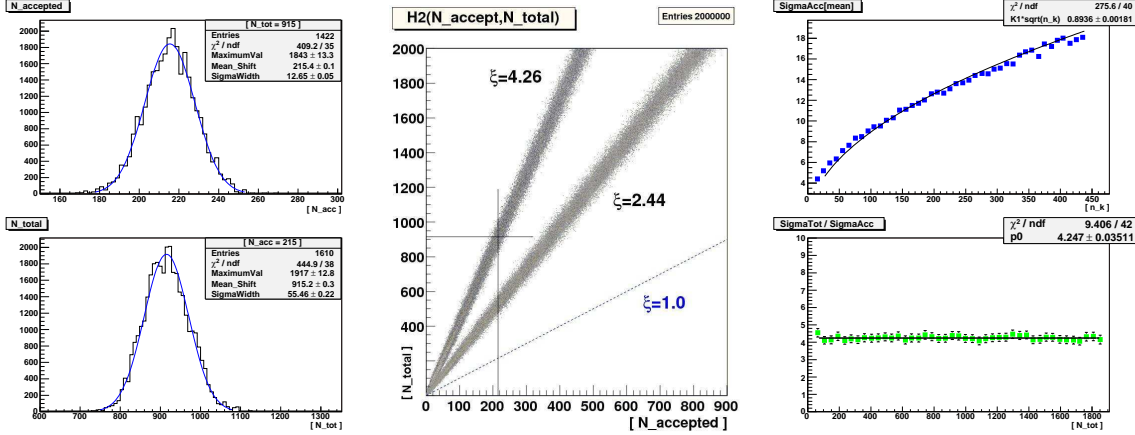


Fig.4: Histograms $H_{\tilde{n}_k}(n_k^{\text{tot}})$, $H_{n_k^{\text{tot}}}(\tilde{n}_k)$, $H2(\tilde{n}_k, n_k^{\text{tot}})$, fitted $\sigma_{\tilde{n}_k} = c \cdot \sqrt{n_k}$ and ratio $\sigma_{\text{tot}}/\sigma_{\tilde{n}_k}$.

VII. FIXED-MULTIPLICITY CORRECTIONS IN REAL EXPERIMENTS

In a general case probability $P(n^{\text{tot}}) \neq \text{const}$ (usually $P(n^{\text{tot}})$ decreases with n^{tot}). The situation is slightly more complex in this case and there appears a shift of average $\langle n^{\text{tot}} \rangle_{\tilde{n}_k}$ (evaluated for events with fixed measured \tilde{n}_k) relative to $\langle \tilde{n}^{\text{tot}} \rangle = \tilde{n}_k \cdot \xi$ expected in the case of $P(n^{\text{tot}}) = \text{const}$. We will show that the shift

$$\delta n^{\text{tot}} = \langle n^{\text{tot}} \rangle_{\tilde{n}_k} - \tilde{n}_k \cdot \xi \quad (34)$$

does not influence fixed-multiplicity correction terms for correlators significantly. Let us assume $P(n^{\text{tot}})$ decreasing with n^{tot} as $a_0 - a_1 \cdot n^{\text{tot}}$ which gives (denoting $a_1/a_0 = \bar{\omega}$) normalized prior probability $P(n^{\text{tot}})$ in the form¹

$$P(n^{\text{tot}}) = 2\bar{\omega}(1 - \bar{\omega} \cdot n^{\text{tot}}) \quad (35)$$

For a given fixed n^{tot} the average $\langle \tilde{n}_k \rangle = n^{\text{tot}}/\xi$ and if $\langle \tilde{n}_k \rangle$ is large enough fluctuations of \tilde{n}_k are close to Gaussian according to Eq.(51). However, average value $\langle n^{\text{tot}} \rangle$ for a given fixed measured \tilde{n}_k is

$$\langle n^{\text{tot}} \rangle_{\tilde{n}_k} = \int n^{\text{tot}} P(n^{\text{tot}} | \tilde{n}_k) dn^{\text{tot}} \quad (36)$$

where $P(n^{\text{tot}} | \tilde{n}_k)$ is a conditional probability distribution of having event with total multiplicity n^{tot} (for events with fixed measured multiplicity \tilde{n}_k). Using prior probability $P(n^{\text{tot}})$ given by Eq.(35) $P(n^{\text{tot}} | \tilde{n}_k)$ can be found analytically if $\xi \tilde{n}_k + 3 \sigma_{\tilde{n}_k} \xi < 1/\bar{\omega}$ (see Appendix)

¹ This probability distribution implies $n^{\text{tot}} < 1/\bar{\omega}$.

$$P(n^{\text{tot}}|\tilde{n}_k) = \frac{e^{-(n^{\text{tot}} - \tilde{n}_k \cdot \xi)^2 / 2\xi^2 \sigma_{\tilde{n}_k}^2}}{\sigma_{\tilde{n}_k} \sqrt{2\pi} \xi} \frac{(1 - n^{\text{tot}} \bar{\omega})}{(1 - \tilde{n}_k \xi \bar{\omega})} \quad (37)$$

and Eq.(36) yields

$$\langle n^{\text{tot}} \rangle_{\tilde{n}_k} = \xi \cdot \tilde{n}_k - \frac{\bar{\omega} \xi^2 \sigma_{\tilde{n}_k}^2}{1 - \tilde{n}_k \xi \bar{\omega}} \quad (38)$$

One can also define *effective* observed multiplicity $\langle \tilde{n}_k \rangle_{\tilde{n}_k} = \langle n^{\text{tot}} \rangle_{\tilde{n}_k} / \xi$ for events with fixed measured \tilde{n}_k

$$\langle \tilde{n}_k \rangle_{\tilde{n}_k} = \tilde{n}_k - \frac{\bar{\omega} \xi \sigma_{\tilde{n}_k}^2}{1 - \tilde{n}_k \xi \bar{\omega}} = \tilde{n}_k - \frac{\omega \sigma_{\tilde{n}_k}^2}{1 - \tilde{n}_k \omega} \quad (39)$$

Using slope parameter ω determined from $P(\tilde{n}_k) = 2\omega(1 - \omega \cdot \tilde{n}_k)$ distribution (accessible experimentally) one has $\bar{\omega} = \omega/\xi$ comparing to Eq.(35) and parameter ξ disappears from Eq.(39). Displacement $\delta \tilde{n}_k = \langle \tilde{n}_k \rangle_{\tilde{n}_k} - \tilde{n}_k$ of the effective multiplicity $\langle \tilde{n}_k \rangle_{\tilde{n}_k}$ relative to fixed measured multiplicity \tilde{n}_k can be relevant e.g. in elliptic flow analysis where effective participant eccentricity $\varepsilon_{\langle \tilde{n}_k \rangle}$ is compared to elliptic flow strength $v_2(\tilde{n}_k)$.

Fixed-multiplicity correction term $\tilde{\Delta}_2^\omega$ for prior probability $P(n^{\text{tot}})$ given by Eq.(35) can be evaluated analogously to Eq.(31)

$$\tilde{\Delta}_2^\omega = \frac{k^2}{\xi^2} \int P(n^{\text{tot}}|\tilde{n}_k) [n^{\text{tot}} - \langle n^{\text{tot}} \rangle_{\tilde{n}_k}]^2 dn^{\text{tot}} \quad (40)$$

which for shifted $\langle n^{\text{tot}} \rangle_{\tilde{n}_k}$ given by Eq.(38) and $P(n^{\text{tot}}|\tilde{n}_k)$ given by Eq.(37) yields

$$\tilde{\Delta}_2^\omega = k^2 \sigma_{\tilde{n}_k}^2 \frac{1 - 2\omega \tilde{n}_k + \omega^2 [\tilde{n}_k^2 - \sigma_{\tilde{n}_k}^2]}{(1 - \omega \tilde{n}_k)^2} \approx k^2 \sigma_{\tilde{n}_k}^2 [1 - \omega^2 \sigma_{\tilde{n}_k}^2] \quad (41)$$

Analytical expression for $\tilde{\Delta}_4^\omega$ can be found using Eq.(26). Assuming $C_2^{\text{true}} = 0$ one has

$$\tilde{\Delta}_4^\omega = 3k^4 \sigma_{\tilde{n}_k}^4 \left[1 - \frac{\sigma_{\tilde{n}_k}^4 \omega^4}{(1 - \tilde{n}_k \omega)^4} - \frac{2\sigma_{\tilde{n}_k}^2 \omega^2}{(1 - \tilde{n}_k \omega)^2} \right] \approx 3k^4 \sigma_{\tilde{n}_k}^4 [1 - 2\omega^2 \sigma_{\tilde{n}_k}^2] \quad (42)$$

If probability $P(n^{\text{tot}}) = \text{const}$ correction term $\tilde{\Delta}_3 \rightarrow 0$ for \tilde{n}_k large. For $\bar{\omega} \neq 0$ which means $P(n^{\text{tot}}) \neq \text{const}$ correction $\tilde{\Delta}_3^\omega$ can be found using Eq.(28):

$$\tilde{\Delta}_3^\omega = -\frac{2k^3 \omega^3 \sigma_{\tilde{n}_k}^6}{(1 - \tilde{n}_k \omega)^3} \approx -2k^3 \omega^3 \sigma_{\tilde{n}_k}^6 \quad (43)$$

For sample of events with constant $P(n^{\text{tot}})$ distribution (which means constant measured $P(\tilde{n}_k)$ distribution) one has $\omega \rightarrow 0$; $\langle n^{\text{tot}} \rangle_{\tilde{n}_k} = \tilde{n}_k \cdot \xi$; $\langle \tilde{n}_k \rangle_{\tilde{n}_k} = \tilde{n}_k$ and $\tilde{\Delta}_n^\omega \rightarrow \tilde{\Delta}_n$.

Since acceptance parameter ξ is not present in Eq.(41,42,43) fixed-multiplicity correction terms can be determined from experimentally accessible quantities: k^2 , ω^2 and $\sigma_{\tilde{n}_k}^2$.

VIII. RELATIONS BETWEEN CORRELATORS

Let us assume that mean transverse momentum \bar{p}_t of particles in events with fixed total multiplicity n^{tot} (e.g. selected from the output of a MC event generator) fluctuates around global mean $\langle \bar{p}_t \rangle = \sum_{k=1}^{N_{ev}} \bar{p}_t^k / N_{ev}$ with probability distribution

$$P(\bar{p}_t^i) = \frac{1}{\sqrt{2\pi} \sigma_{\bar{p}_t}} e^{-(\bar{p}_t^i - \langle \bar{p}_t \rangle)^2 / 2\sigma_{\bar{p}_t}^2} \quad (44)$$

In this case $\sum_{k=1}^{N_{ev}} (\bar{p}_t^k - \langle \bar{p}_t \rangle)^2 / N_{ev} = \sigma_{\bar{p}_t}^2$ and one obtains

$$C_2^{\text{calc}} = C_2^{\text{true}} + \sigma_{\bar{p}_t}^2 \quad (45)$$

Thus C_2^{calc} correlator contains $\sigma_{\bar{p}_t}^2$ contribution from event-by-event fluctuations of observable mean \bar{p}_t and C_2^{true} contribution from genuine two-particle correlations. This can be verified directly using a suitable MC event generator. For fluctuations of observable mean given by Eq.(44) one has $\sum_{k=1}^{N_{ev}} (\bar{p}_t^k - \langle \bar{p}_t \rangle)^4 / N_{ev} \approx 3 \sigma_{\bar{p}_t}^4$ and in agreement with Eq.(21)

$$C_4^{\text{calc}} = C_4^{\text{true}} + 3 \sigma_{\bar{p}_t}^4 + 6 \sigma_{\bar{p}_t}^2 C_2^{\text{true}} \quad (46)$$

Assuming $C_4^{\text{true}} \rightarrow 0$ one can try to separate $\sigma_{\bar{p}_t}^2$ and C_2^{true} contributions as

$$(C_2^{\text{true}})^2 = (C_2^{\text{calc}})^2 - C_4^{\text{calc}} / 3 \quad (47)$$

which is a solution of Eqs.(45,46) for vanishing 4-particle correlations ($C_4^{\text{true}} = 0$).

IX. CONCLUSIONS

A simple analytical calculation has shown that systematical shifts $\tilde{\Delta}_n$ in calculated values of correlators are generated if observable mean of the quantity under study is multiplicity-dependent $\bar{x}(n_k) \neq \text{const.}$ One can subtract such systematical effects from the calculated correlators C_n^{calc} to obtain "true" correlators using: $C_n^{\text{true}} = C_n^{\text{calc}} - \tilde{\Delta}_n$.

X. APPENDIX

Fixed-multiplicity correction term given by Eq.(32) contains quantity $\sigma_n(\xi, \tilde{n}_k)$ to be determined from MC simulation. We will show that width $\sigma_n(\xi, \tilde{n}_k)$ of n_i^{tot} fluctuations at

given fixed measured \tilde{n}_k^* can be related to the width $\sigma_{\tilde{n}_k}$ of \tilde{n}_k fluctuations at fixed $n_*^{\text{tot}} \approx \xi \cdot \tilde{n}_k^*$ as $\sigma_n(\xi, \tilde{n}_k^*) = \xi \cdot \sigma_{\tilde{n}_k}(n_*^{\text{tot}})$ where ξ is acceptance parameter $\xi = n^{\text{tot}} / \langle \tilde{n}_k \rangle$ and the width $\sigma_{\tilde{n}_k}$ can be expressed as $\sigma_{\tilde{n}_k}(n_*^{\text{tot}}) = \langle \tilde{\sigma} \rangle \sqrt{\tilde{n}_k^*}$.

To demonstrate this let us divide given detector acceptance Ω_ξ into N acceptance subregions $\omega_1 + \omega_2 + \dots + \omega_N = \Omega_\xi$ in such a way that in every acceptance region ω_i equal average number of particles $\langle \tilde{n}^{\omega_i} \rangle = \langle \tilde{n}^{\omega_j} \rangle = \langle \tilde{n}_k \rangle / N$ will be measured. Using $N = \tilde{n}_k$ acceptance subregions one has $\langle \tilde{n}^{\omega_i} \rangle \approx 1$ and $\sum_{i=1}^N \langle \tilde{n}^{\omega_i} \rangle = \langle \tilde{n}_k \rangle$. For events with fixed total multiplicity n^{tot} and average measured multiplicity $\langle \tilde{n}_k \rangle$ the number of particles $\tilde{n}_k^{\omega_i}$ observed in acceptance region ω_i will fluctuate (event-by-event) with some probability distribution $P_{\omega_i}(n_k^{\omega_i})$ characterized by the mean $\langle \tilde{n}^{\omega_i} \rangle$ and variance σ_{ω_i} .

Assuming that probability distributions $P_{\omega_i}(n_k^{\omega_i})$ satisfy conditions for the applicability of generalized (e.g. m -dependent) Central Limit Theorem one can write

$$P_{\Omega_\xi}(\tilde{n}_k) = P_{\Omega_\xi}\left(\sum_{i=1}^N \tilde{n}_k^{\omega_i}\right) \approx \frac{e^{-(\tilde{n}_k - \langle \tilde{n}_k \rangle)^2 / 2\sigma_{\tilde{n}_k}^2}}{\sqrt{2\pi\sigma_{\tilde{n}_k}^2}} = P_{n^{\text{tot}}}(\tilde{n}_k) = P(\tilde{n}_k | n^{\text{tot}}) \quad (48)$$

where $\sigma_{\tilde{n}_k} \approx \sqrt{\sigma_{\omega_1}^2 + \sigma_{\omega_2}^2 + \dots + \sigma_{\omega_N}^2} = \sqrt{N} \langle \tilde{\sigma} \rangle$ (denoting $(\sum_{i=1}^N \sigma_{\omega_i}^2 / N)^{1/2} = \langle \tilde{\sigma} \rangle$). This suggests that for $\langle \tilde{n}_k \rangle = n^{\text{tot}} / \xi$ large enough the probability distribution of events with measured multiplicities \tilde{n}_k in the set of events with fixed total multiplicity n^{tot} tends to be Gaussian and its width $\sigma_{\tilde{n}_k}$ increases with measured multiplicity as

$$\sigma_{\tilde{n}_k} = c \cdot \sqrt{\langle \tilde{n}_k \rangle} \quad (49)$$

One can ask a similar question in the other way around: What are the fluctuations of total multiplicity of particles n_i^{tot} for events with fixed observed multiplicity \tilde{n}_k ?

Using the Bayes' theorem [6] one can calculate probability $P(n^{\text{tot}} | \tilde{n}_k)$ of observing the event with total multiplicity n^{tot} in the subset of events with fixed measured multiplicity \tilde{n}_k

$$P(n^{\text{tot}} | \tilde{n}_k) = \frac{P(\tilde{n}_k | n^{\text{tot}}) P(n^{\text{tot}})}{\int P(\tilde{n}_k | n^{\text{tot}}) P(n^{\text{tot}}) dn^{\text{tot}}} \quad (50)$$

where $P(A|B)$ denotes a conditional probability of observing quantity A for given B and $P(n^{\text{tot}})$ is the prior probability² of event with total multiplicity n^{tot} . Choosing the sample of events with $P(n^{\text{tot}}) = \text{const}$ simplifies the situation and one has $P(n^{\text{tot}} | \tilde{n}_k) = \lambda \cdot P(\tilde{n}_k | n^{\text{tot}})$ where λ is a normalization constant.

² Note, that $P(n^{\text{tot}})$ depends on a particular setting of the detector trigger.

For $\langle \tilde{n}_k \rangle$ large enough $P(\tilde{n}_k | n^{\text{tot}})$ given by Eq.(48) can be used to obtain probability $P(n^{\text{tot}} | \tilde{n}_k)$ of having event with total multiplicity n^{tot} in the group of events with fixed measured multiplicity \tilde{n}_k . For event sample with probability $P(n^{\text{tot}}) = \text{const}$ one has:

$$P(n^{\text{tot}} | \tilde{n}_k) = \lambda \cdot P(\tilde{n}_k | n^{\text{tot}}) = \lambda \frac{e^{-(\xi \cdot \tilde{n}_k - \xi \langle \tilde{n}_k \rangle)^2 / 2\xi^2 \sigma_{\tilde{n}_k}^2}}{\sqrt{2\pi \sigma_{\tilde{n}_k}^2}} = \frac{e^{-(n^{\text{tot}} - \tilde{n}_k \cdot \xi)^2 / 2\sigma_{\text{tot}}^2}}{\sqrt{2\pi \sigma_{\text{tot}}^2}} \quad (51)$$

where we have denoted $\xi \cdot \sigma_{\tilde{n}_k} = \sigma_{\text{tot}}$, used $n^{\text{tot}}/\xi = \langle \tilde{n}_k \rangle$ and normalized the resulting Gaussian distribution to unity. For event sample with probability $P(n^{\text{tot}}) = 2\bar{\omega}(1 - \bar{\omega} n^{\text{tot}})$ evaluation of denominator in Eq.(50) gives $2\xi\bar{\omega}(1 - \xi\tilde{n}_k\bar{\omega})$ and Eq.(50) then yields

$$P(n^{\text{tot}} | \tilde{n}_k) = \frac{e^{-(n^{\text{tot}} - \tilde{n}_k \cdot \xi)^2 / 2\xi^2 \sigma_{\tilde{n}_k}^2}}{\sigma_{\tilde{n}_k} \sqrt{2\pi} \xi} \frac{(1 - n^{\text{tot}}\bar{\omega})}{(1 - \tilde{n}_k \xi \bar{\omega})} \quad (52)$$

which is valid for fluctuations of n^{tot} (at given fixed \tilde{n}_k) within the range of $P(n^{\text{tot}})$ distribution. For $P(n^{\text{tot}}) = 2\bar{\omega}(1 - \bar{\omega} n^{\text{tot}})$ this means $\tilde{n}_k \xi + 3\sigma_{\text{tot}} < 1/\bar{\omega}$ which keeps $\xi \tilde{n}_k \bar{\omega} < 1$.

For Poissonian probability $P(\tilde{n}_k) = \langle \tilde{n}_k \rangle^{\tilde{n}_k} \cdot e^{-\langle \tilde{n}_k \rangle} / \tilde{n}_k!$ of measuring \tilde{n}_k particles in a detector (exposed to events with fixed multiplicity $n^{\text{tot}} = \xi \langle \tilde{n}_k \rangle$) Bayes' law gives asymmetrical probability of n^{tot} for a given fixed \tilde{n}_k :

$$\tilde{P}(n^{\text{tot}} | \tilde{n}_k) = \lambda \frac{(n^{\text{tot}}/\xi)^{\tilde{n}_k} \cdot e^{-n^{\text{tot}}/\xi}}{\tilde{n}_k!} P(n^{\text{tot}}) = \lambda' \cdot e^{-n^{\text{tot}}/\xi} \cdot (n^{\text{tot}})^{\tilde{n}_k} \cdot P(n^{\text{tot}}) \quad (53)$$

(here λ and λ' are normalization factors). Fluctuations of n^{tot} at small fixed \tilde{n}_k can be approximated by this function (see Fig.3 for $\tilde{P}(n^{\text{tot}} | \tilde{n}_k)$ at $\tilde{n}_k = 10$ using $P(n^{\text{tot}}) = \text{const}$).

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